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EXTENSION OF PLANE-WAVE SCATTERING-MATRIX THEORY
OF ANTENNA - ANTENNA INTERACTIONS TO THREE ANTENNAS:
A NEAR-FIELD RADAR CROSS SECTION CONCEPT

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1. ## Abstract

This paper presents a three-antenna plane-wave scattering-matrix (PWSM) formulation and a formal solution. An example will be demonstrated in which two of the three antennas are electromagnetically identical (the transmitter and receiver) and the third (the scatterer) has arbitrary electromagnetic properties. A reduced reflection integral-matrix will be discussed which describes the transmit, scatter, receive (TSR) interaction. An antenna scatterer spectral tensor Greens function is identified. In this formulation the transmit spectrum will be scattered by the third arbitrary antenna (target) and this scattered spectrum may be considered to have originated from a transmitting antenna. Near-field antenna measurement techniques are applicable which determine the electric (scattered) field spectral density function.[1, 3] If a second deconvolution is applied, a transmit probe corrected spectral density function or scattering tensor can be determined in principle. In either case, a near- or far-electric field can be calculated and a radar cross section determined.

2.   Introduction

The successful results that near-field antenna measurement techniques have achieved in determining far-field antenna patterns encourage the idea that perhaps similar techniques may be applied to determine a target's far-field radar cross-section (RCS) based upon near- scattered field measurements. The idea that a target's far-field RCS can be determined from near-scattered field measurements is considered here as a near-field RCS concept. Planar scanning near-field antenna measurement is theoretically substantiated using the two-antenna, plane-wave scattering-matrix (PWSM) formulation.[2] Since determining an RCS requires a transmit probe, a target, and a receive probe, extending the PWSM formulation to include three antennas was chosen for investigating the feasibility of a near-field RCS concept. This paper presents a three-antenna PWSM formulation, discusses some of the results, and shows how this formulation substantiates the near-field RCS concept.

In the following section, the theory and definitions associated with the PWSM fomulation are stated.[2] This is followed by a section on the three-antenna PWSM equations and a general solution. A specific solution for the RCS problem (TSR interaction) is then presented in section 5. The paper closes by stating conclusions and identifying future efforts related to this topic.

3. Background

The PWSM formulation of antennas depends upon the plane-wave representation of an electromagnetic field. Beginning with plane-wave solutions to Maxwell's equations in a rectangular xyz coordinate system in free space, solutions for $\bar{E}$ and $\bar{H}$ can be constructed from elementary plane-waves. Referring to figure 1, an antenna system bounded by two planar surfaces F_1 and F_2, which are transverse to the z-axis direction $(\hat{e}_z)$, can have corresponding $\bar{E}$ and $\bar{H}$ field solutions in the form of weighed-sums of plane-waves traveling to the right and left on either side of the antenna. Specifically,

$$\bar{E}_{qt}(\bar{r}) = \frac{1}{2\pi} \int \sum_m \left[b_q(m,\bar{K}) e^{\pm i \gamma z} + a_q(m,\bar{K}) e^{\mp i \gamma z} \right] \hat{\kappa}_m \, e^{i\bar{K}\cdot\bar{R}} \, d\bar{K} \qquad (1\text{-}a)$$

$$\bar{B}_{qt}(\bar{r}) = \frac{1}{2\pi} \int \sum_m \left[b_q(m,\bar{K}) e^{\pm i \gamma z} - a_q(m,\bar{K}) e^{\mp i \gamma z} \right] \hat{n}_q \times \hat{\kappa}_m \, n_m(\bar{K}) e^{i\bar{K}\cdot\bar{R}} \, d\bar{K} \qquad (1\text{-}b)$$

where the following sign conventions and definitions apply: q takes on values of 1 or 2 which correspond to the right (F_1) or left (F_2) side of the antenna, respectively; also, q values of 1 or 2 dictate use of the upper or lower sign respectively, found with the z-dependent exponentials; the subscript t denotes the transverse components (w.r.t. $\hat{e}_z$); m takes on values of 1 and 2 which correspond to TM and TE polarizations respectively; the transverse vectors $\bar{K}$ and $\bar{R}$ and the transverse unit vector $\hat{\kappa}_m$ and γ are defined in (1-c); $\hat{n}_q$ is the unit outward normal to F_q;

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Figure 1. Some Notation for Plane-Wave Scattering-Matrix Description of Antennas

$\eta_m(\bar{K})$ are the TM and TE wave admittances; $b_q(m,\bar{K})$ and $a_q(m,\bar{K})$ are the continuous spectral (angular) density functions for emergent and incident (relative to $\hat{n}_q$ and F_q) plane-waves, respectively. Finally, an $e^{-i\omega t}$ time dependence is used.

$$\bar{k} = k_x\hat{e}_x + k_y\hat{e}_y + k_z\hat{e}_n \qquad ; \qquad \bar{r} = r_x\hat{e}_x + r_y\hat{e}_y + r_z\hat{e}_z$$

$$\bar{K} = k_x\hat{e}_x + k_y\hat{e}_y \qquad ; \qquad \bar{R} = r_x\hat{e}_x + r_y\hat{e}_y$$

$$k_z = \pm\sqrt{k^2 - K^2} = \pm\gamma \qquad ; \qquad \hat{\kappa}_1 = \bar{K}/|\bar{K}| \qquad (1\text{-c})$$

$$k^2 = \omega^2\epsilon_o\mu_o \qquad ; \qquad \hat{\kappa}_2 = \hat{e}_z \times \hat{\kappa}_1$$

The remaining component of $\bar{E}$ and $\bar{H}$ $(\hat{e}_z)$ can be obtained by using the fact that each elementary plane-wave is orthogonal to $\bar{k}$, i.e., $\bar{k}\cdot\bar{E}(\text{or } \bar{H}) = 0$. This is referred to as transversality in reference 1.

Expressions for $b_q(m,\bar{K})$ and $a_q(m,\bar{K})$ are obtained by inverting (1-a,-b).

$$b_q(m,\bar{K}) = \frac{e^{\mp i\gamma z}}{4\pi}\,\hat{\kappa}_m\cdot\int\left\{\bar{E}_q(\bar{R},z) + \eta_m^{-1}\,\bar{H}_q(\bar{R},z)\times\hat{n}_q\right\}e^{-i\bar{K}\cdot\bar{R}}\,d\bar{R} \qquad (2\text{-a})$$

$$a_q(m,\bar{K}) = \frac{e^{\pm i\gamma z}}{4\pi}\,\hat{\kappa}_m\cdot\int\left\{\bar{E}_q(\bar{R},z) - \eta_m^{-1}\,\bar{H}_q(\bar{R},z)\times\hat{n}_q\right\}e^{-i\bar{K}\cdot\bar{R}}\,d\bar{R} \qquad (2\text{-b})$$

Note from (2) that a knowledge of $\bar{E}$ and $\bar{H}$ in the transverse plane $\bar{R}$ is sufficient to determine the spectral density functions and therefore $\bar{E}$ and $\bar{H}$ anywhere[1] using (1-a,-b) and transversality.

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1 If $\bar{E}$ and $\bar{H}$ are determined in a transverse plane which excludes evanescent modes, application of (1-a,-b) in the reactive near-field would be erroneous. Therefore, constraints used in (2) become restrictions in (1-a,-b).

These equations and definitions show how the spectral density functions are related to the $\bar{E}$ and $\bar{H}$ field. The spectral density functions are also used explicitly in the scattering-matrix fomulation of antenna characteristics and in fact will be defined in terms of the scattering parameters. Therefore, the dependence of $\bar{E}$ and $\bar{H}$ upon the scattering-matrix parameters will be made apparent. Also, these equations will be used to derive a set of joining-equations for the three-antenna formulation which relates the incident spectral density functions of one antenna to the emergent spectral density functions of another antenna.

The scattering matrix parameters will now be defined in conjunction with the spectral density functions $a_q(m,\bar{K})$ and $b_q(m,\bar{K})$. Since an antenna is an imperfect receiver, an incident electromagnetic wave with the corresponding incident spectral density function $a_p(n,\bar{L})$ will be scattered. The subscript p accounts for the fact that the incident spectral density function can impinge upon the antenna from the right side (p=1) or the left side (p=2). For each particular transverse incident vector $\bar{L}$ and a particular polarization n any scattered direction $\bar{K}$ and polarization m is, in general, possible and may be in either the forward or backward direction (q = 1 or 2). The emergent spectral density function $b_q(m,\bar{K})$ is then dependent upon the incident wave by the following equation:

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$$b_q(m,\bar{K}) = \sum_p \int_{\bar{L}} \sum_n S_{qp}(m,\bar{K};n,\bar{L}) \; a_p(n,\bar{K}) \; d\bar{L} \qquad (3)$$

where $S_{qp}(m,\bar{K};n,\bar{L})$ is an element of a dyadic scattering tensor. If the antenna is in an active (transmit) mode then $b_q(m,\bar{K})$ will have an additional spectral contribution which is the product of the antenna waveguide feed modal amplitude a_0 and the antenna transmitting (spectral) characteristics $S_{qo}(m,\bar{K})$. To complete the scattering-matrix description of the antenna characteristics, the antenna waveguide feed emergent modal amplitude b_0 will be due to two factors: the product of a_0 and the antenna waveguide impedance mismatch S_{oo}; and the product of the incident wave spectral amplitude $a_p(n,\bar{K})$ and the antenna receiving spectral characteristics $S_{oq}(m,\bar{K})$. The antenna scattering-matrix equations can now be written as:

$$b_o = S_{oo}a_o + \sum_p \int_{\bar{L}} \sum_n S_{op}(n,\bar{L}) \; a_p(n,\bar{L}) \; d\bar{L}$$

$$b_q(m,\bar{K}) = S_{qo}(m,\bar{K}) \, a_o + \sum_p \int \sum_n S_{qp}(m,\bar{K};n,\bar{L}) \; a_p(n,\bar{L}) \; d\bar{L}$$

$$(4)$$

Each of the scattering parameters can be vectorized by using the TM and TE unit vectors $\hat{\kappa}_1$ and $\hat{\kappa}_2$. The invariance of (4) with respect to the choice of coordinates in the transverse plane should be noted. All of the necessary antenna scattering parameters and formulations have been stated. The three antenna scattering equations will now be considered.

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4. Three-Antenna Formulation

Figure 2 shows the three-antenna configuration to be studied. A superscript will be added to all of the PWSM parameters to indicate the antenna to which a parameter corresponds (1, 2, or 3). Antennas 1 and 2 are located in the same transverse plane. Each antenna will have its own relative coordinate system and therefore its own relative incident and emergent spectral density functions.

Antenna 1 of figure 2 will be the reference. The other antennas will have corresponding incident and emergent spectral density functions relative to antenna 1. These relations (joining-equations) are developed using the geometry in figure 2, equations (2) and the uniqueness of $\ddot{E}$ and $\bar{H}$ at any physical point. These joining-equations are:

$$\bar{a}_1^2 = \bar{T}\bar{a}_1^1 ; \quad \bar{b}_1^2 = \bar{T}\bar{b}_1^1 ; \quad \bar{T} = \bar{I}e^{i\bar{K}\cdot\bar{Q}} ; \quad \bar{T}^{-1} = \bar{I}e^{-i\bar{K}\cdot\bar{Q}} \qquad (5\text{-}a)$$

$$\bar{b}_2^3 = \bar{T}_{-1}\bar{a}_1^1 = \bar{T}_{-2}\bar{a}_1^2 ; \quad \bar{a}_2^3 = \bar{T}_{+1}\bar{b}_1^1 = \bar{T}_{+2}\bar{b}^2 \qquad (5\text{-}b)$$

$$\bar{T}_{\substack{-1 \\ +2}} = \bar{I}\,e^{i\bar{k}^{\mp}\cdot\,\bar{r}_{\substack{1 \\ 2}}} \qquad (5\text{-}c)$$

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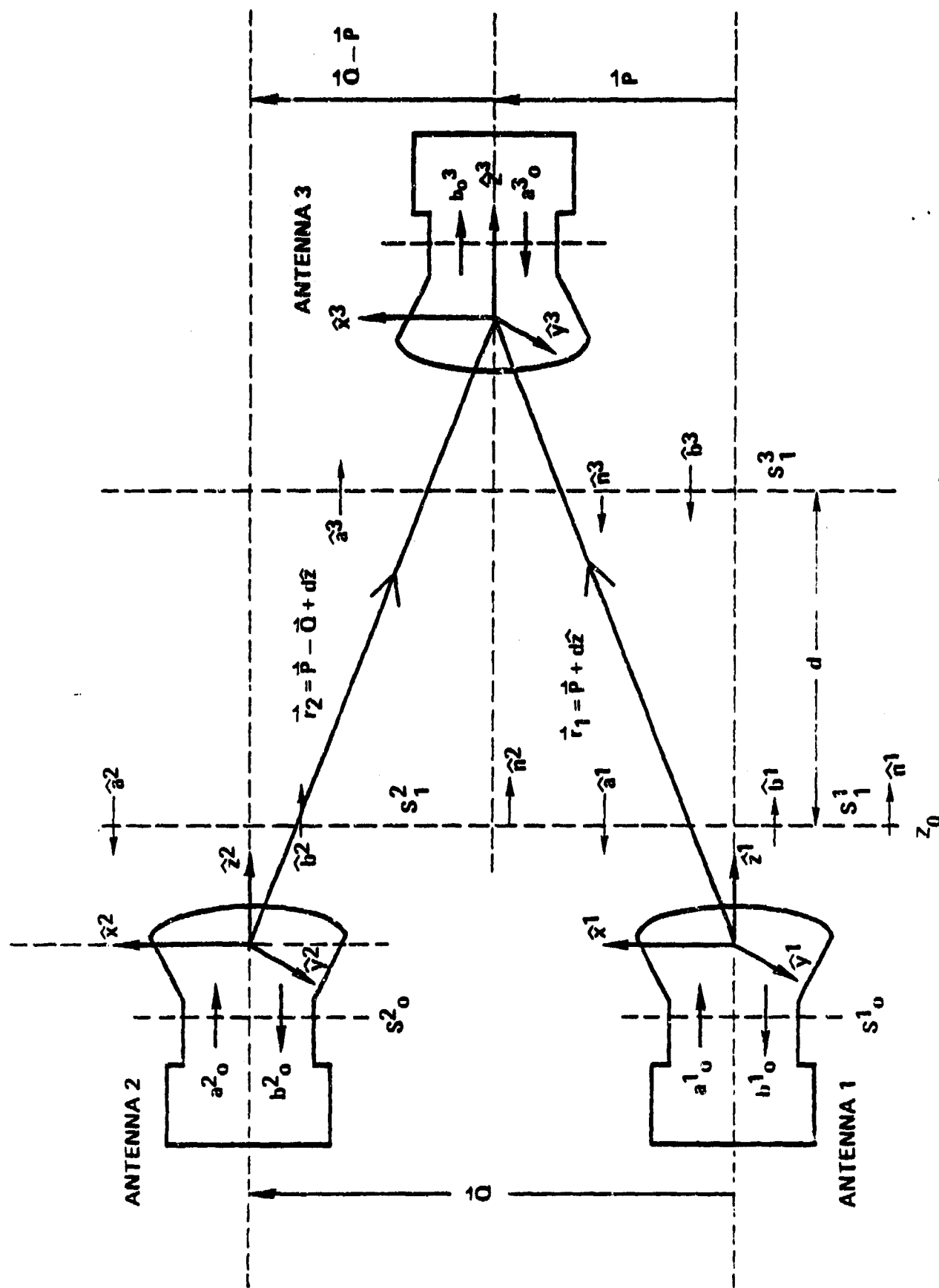
Figure 2. Three Antenna Scattering Configuration

where $\overline{\overline{1}}$ is the 2 x 2 identity matrix and the $\overline{a}$ and $\overline{b}$ spectral density function vectors have two components, one for each polarization, TM and TE (i.e., $\hat{\kappa}_1$ and $\hat{\kappa}_2$). Similarly, using vector-operator notation the three-antenna coupled scattering equations are:

	input reflection	self receive characteristics	mutual receive characteristics	
b_0^1 =	$s_{00}^1 \, a_0^1$ +	$\overline{s}_{01} \, \overline{a}_1^1$ +	$\overline{s}_{01}^{12} \, \overline{a}_1^2$	(6-a)

	transmit characteristics	mutual transmittance	self scatter	mutual scatter
$\overline{b}_1^1$ =	$\overline{s}_{10}^1 \, a_0^1$ +	$\overline{s}_{10}^{12} \, a_0^2$ +	$\overline{s}_{11}^1 \, \overline{a}_1^1$ +	$\overline{s}_{11}^{12} \, \overline{a}_1^2$

$$b_0^2 = s_{00}^2 \, a_0^2 + \overline{s}_{02}^2 \, \overline{a}_1^2 + \overline{s}_{02}^{21} \, \overline{a}_1^1 \quad (6b)$$

$$\overline{b}_1^2 = \overline{s}_{10}^2 \, a_{10}^2 + \overline{s}_{10}^{21} \, a_0^2 + \overline{s}_{11}^2 \, \overline{a}^2 + \overline{s}_{11}^{21} \, \overline{a}_1^1$$

$$b_0^3 = s_{00}^3 \, a_0^3 + \overline{s}_{02}^3 \, \overline{a}_2^3$$

$$(6-c)$$

$$\overline{b}_2^3 = \overline{s}_{20}^3 \, a_0^3 + \overline{s}_{22}^3 \, \overline{a}_2^3$$

These scattering equations are coupled due to the mutual interactions of antenna 1 and 2. Note that the subscripts on the vector or tensor quantities refer to the right or left side of a

particular antenna as defined under (2). This being understood, the subscripts on $\bar{a}$ and $\bar{b}$ will now be suppressed. The mutual receive parameters , $\bar{S}_{01}^{12}$ and $\bar{S}_{01}^{21}$, account for the presence of a second antenna in the transverse plane located at $z_1 = z_2 = 0$ of figure 2.

Two mutual interactions are possible. The first is one antenna transmitting in an active mode and the second antenna directly receiving this primary radiation. This interaction is encountered in antenna array theory and can be accounted for in S_{oo} as an active input impedance. The second interaction is due to radiation being scattered from one antenna and received by the second. This is explicity accounted for in the scattering equations as the mutual receive parameters $\bar{S}_{01}^{12}$ and $\bar{S}_{01}^{21}$. The mutual receive parameter $\bar{S}_{01}^{12}$ is due to $\bar{a}^2$ being scattered from antenna 2 and received by antenna 1.

Using the two-antenna solution (located on the same transverse plane) and (5-a), $\bar{S}_{01}^{12}$ can be expressed as:

$$\bar{S}_{01}^{12} = \bar{S}_{01}^{1} \; \bar{T}^{-1} \; \bar{S}_{11}^{2} \tag{7}$$

Similarly, the mutual receive parameter $\bar{S}_{01}^{21}$ can be expressed as:

$$\bar{S}_{01}^{21} = \bar{S}_{02}^{2} \; \bar{T} \; \bar{S}_{11}^{1} \tag{8}$$

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The mutual transmittances $\bar{S}_{10}^{12}$ and $\bar{S}_{10}^{21}$ account for primary radiation being transmitted from one antenna, transformed (or propagated) to another antenna reference, and scattered. As in (7) and (8), $\bar{S}_{10}^{12}$ can be expressed as:

$$\bar{S}_{10}^{12} = (\bar{I} + \bar{S}_{11}^{1})\ \bar{T}^{-1}\ \bar{S}_{10}^{2} \qquad (9)$$

Similarly, $\bar{S}_{10}^{21}$ can be expressed as:

$$\bar{S}_{10}^{21} = (\bar{I} + \bar{S}_{11}^{2})\ \bar{T}\ \bar{S}_{10}^{1} \qquad (10)$$

The final parameters to be defined under (6) are the mutual scattering parameters $\bar{S}_{11}^{12}$ and $\bar{S}_{11}^{21}$. Mutual scattering accounts for one antenna scattering radiation, transformed to another antenna reference. These are expressed as in (11) and (12).

$$\bar{S}_{11}^{12} = \bar{T}^{-1}\ \bar{S}_{11}^{2} \qquad (11)$$

$$\bar{S}_{11}^{21} = \bar{T}\ \bar{S}_{11}^{1} \qquad (12)$$

The mutual transmit and scattering parameters are necessary for $\bar{b}^1$ or $\bar{b}^2$ to separately represent all interactions or processes contributing to rightward-traveling spectral radiation. A formal solution to (6) can be derived where the antenna wave guide feed emergent modal amplitudes b_0 are written in terms of the antenna wave guide feed exiting amplitudes a_0. Using (5) and (7) - (12) in (6) the following matrix solution can be obtained:

$$
\begin{pmatrix} b_0^1 \\ b_0^2 \\ b_0^3 \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{pmatrix} \begin{pmatrix} a_0^1 \\ a_0^2 \\ a_0^3 \end{pmatrix} \tag{13}
$$

$$
M_{11} = S_{00}^1 + \bar{S}_{01}^1 (\bar{I} + \bar{R}_{11}^2)\,[\bar{I} - \bar{R}_{11}^{13}\,(\bar{S}_{11}^1 + \bar{R}_{11}^2)]^{-1}\;\bar{R}_{22}^{13}\;\bar{S}_{10}^1 \tag{13-a}
$$

$$
M_{12} = \bar{S}_{01}^1\,(\bar{I} + \bar{R}_{11}^2)\,[\bar{I} - \bar{R}_{22}^{13}\,(\bar{S}_{11}^1 + \bar{R}_{11}^2)]^{-1}\;\bar{R}_{22}^{13}\;\bar{S}_{10}^{12} \tag{13-b}
$$

$$
M_{13} = \bar{S}_{01}^1\,(\bar{I} + \bar{R}_{11}^2)\,[\bar{I} - \bar{R}_{22}^{13}\,(\bar{S}_{11}^1 + \bar{R}_{11}^2)]^{-1}\;\bar{T}_{-1}^{-1}\;\bar{S}_{20}^3 \tag{13-c}
$$

$$
M_{21} = \bar{S}_{01}^2 (\bar{I} + \bar{R}_{11}^1)\,[\bar{I} - \bar{R}_{22}^{23}\,(\bar{S}_{11}^2 + \bar{R}_{11}^1)]^{-1}\;\bar{R}_{22}^{23}\;\bar{S}_{10}^{21} \tag{13-d}
$$

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$$M_{22} = S_{00}^2 + \bar{S}_{01}^2 \, (\bar{I} + \bar{R}_{11}^1) \, [\bar{I} - \bar{R}_{22}^{23} (\bar{S}_{11}^2 + \bar{R}_{11}^1)]^{-1} \, \bar{R}_{22}^{23} \, \bar{S}_{10}^2 \qquad \text{(13-e)}$$

$$M_{23} = \bar{S}_{01}^2 \, (\bar{I} + \bar{R}_{11}^1) \, [\bar{I} - \bar{R}_{22}^{23} (\bar{S}_{11}^2 + \bar{R}_{11}^1)]^{-1} \, \bar{T}_{-2}^{-1} \, \bar{S}_{20}^3 \qquad \text{(13-f)}$$

$$M_{31} = \bar{S}_{02}^3 \, [\bar{I} - \bar{R}_{11}^3 \, \bar{S}_{22}^3]^{-1} \, \bar{T}_{+1} \, \bar{S}_{10}^1 \qquad \text{(13-g)}$$

$$M_{32} = \bar{S}_{02}^3 \, [\bar{I} - \bar{R}_{11}^3 \, \bar{S}_{22}^3]^{-1} \, \bar{T}_{+1} \, \bar{S}_{10}^{12} \qquad \text{(13-h)}$$

$$M_{33} = S_{00}^3 + \bar{S}_{02}^3 \, [\bar{I} - \bar{R}_{22}^3 \, \bar{S}_{22}^3]^{-1} \, \bar{R}_{11}^3 \, \bar{S}_{20}^3 \qquad \text{(13-i)}$$

where the following 2 x 2 matrices $\bar{\bar{R}}$ are given by:

$$\bar{R}_{11}^1 = \bar{T}\bar{S}_{11}^1\bar{T}^{-1}; \qquad \bar{R}_{11}^2 = \bar{T}^{-1}\bar{S}_{11}^2\bar{T}; \qquad \bar{R}_{11}^3 = \bar{T}_{+1}(\bar{S}_{11}^1 + \bar{R}_{11}^2) \, \bar{T}_{-1}^{-1}$$

$$\bar{R}_{22}^{13} = \bar{T}_{-1}^{-1} \, \bar{S}_{22}^3\bar{T}_{+1}; \qquad \bar{R}_{22}^{23} = \bar{T}_{-2}^{-1} \, \bar{S}_{22}^3\bar{T}_{+2} \qquad \text{(14)}$$

The $\bar{\bar{R}}$ matrices represent transformations of the scattering para-
meters among the antennas. The expressions contained in (13)
account for the following interactions: direct transmit and
receive (zero-order); transmit, scatter and receive (TSR)

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(first-order); and higher order scattering among the three antennas. In obtaining (13) expressions for $\bar{a}^1$, $\bar{b}^1$, $\bar{a}^2$, $\bar{b}^2$, and $\bar{a}^3$, $\bar{b}^3$ are found which can be used in (2) for explicitly representing the corresponding $\bar{E}$ and $\bar{H}$ fields.

5. Transmit-Scatter-Receive Interaction: The Radar Problem

For simulating the radar problem, let antenna 1 transmit, antenna 3 be passively scattering radiation, and antenna 2 be operating in the receiving mode. Further, let antennas 1 and 2 have identical characteristics (as defined under (3) and (4)) denoted by

$$\bar{s}_{01}^1 = \bar{s}_{01}^2 = \bar{I}_{01}, \qquad \bar{s}_{10}^1 = \bar{s}_{10}^2 = \bar{I}_{10}, \qquad \bar{s}_{11}^1 = \bar{s}_{11}^2 = \bar{I}_{11} \qquad (15)$$

If mutual scattering between antennas 1 and 2 is assummed negligible, then since $a_0^2 = a_0^3 = 0$, b_0^2 from (13) becomes:

$$b_0^2 = \bar{I}_{01} \; [\bar{I} - \bar{\bar{R}}_{22}^{23} \; \bar{\bar{I}}_{11}^2]^{-1} \; \bar{\bar{R}}_{22}^{23} \; \bar{\bar{T}} \; \bar{I}_{10}^1 \; a_0^1 \qquad (16)$$

Considering the first reflection to be dominant and subsequent reflections to be negligible, (16) can be further reduced to:

$$b_0^2 = \bar{I}_{01} \ \bar{T}_{-2}^{-1} \ \bar{S}_{22}^3 \ \bar{T}_{+1} \ \bar{I}_{10}^1 \ a_0^1 \qquad (17)$$

In explicit integral form, (17) can be written as

$$b_0^2(\bar{r}_1,\bar{r}_1) = a_0^1 \int [\bar{I}_{01}(\bar{K})e^{-i\bar{K}\cdot\bar{r}_2} \cdot \int \bar{S}_{22}^3(\bar{K},\bar{L})\cdot\bar{I}_{10}(\bar{L})e^{i\bar{L}\cdot\bar{r}_1} \, d\bar{L}] \, d\bar{K} \qquad (18)$$

receiving characteristics   right to left propagation   spectral dyadic Greens Function   transmitting characteristics   left to right propagation

and is a reduced reflection integral. If multiple reflections are considered significant, (18) can be used as a first approximation to b_0^2, and using an appropriate iterative technique b_0^2 may be evaluated in principle if (16) is convergent. For this latter class of problems it may be possible to evaluate the full "unreduced" form for b_0^2 ($a_0^2 = a_0^3 = 0$) as written in (13-d).

The reduced reflection integral (18) mathematically describes a TSR interaction which is the radar problem. To make apparent the similarity of (18) with the transmission integral (reference 1) let

$$\bar{I}_{10}'(\bar{r}_1,\bar{K}) = \int \bar{\bar{S}}_{22}^3 \ (\bar{K},\bar{L})\cdot\bar{I}_{10}(\bar{L}) \ e^{i\bar{L}\cdot\bar{r}_1} \, d\bar{L} \qquad (19)$$

Then (18) becomes:

$$b_0^2 (\bar{r}_1, \bar{r}_2) = a_0^1 \int \bar{\bar{I}}_{01}(\bar{K}) \cdot \bar{\bar{I}}_{10}^{\prime}(\bar{r}_1, \bar{K}) \; e^{-i\bar{k}^- \cdot \bar{r}_2} \; d\bar{K} \qquad (20)$$

which is indeed a transmission integral. The transmit antenna has a spectral radiation pattern $\bar{I}_{10}^{\prime}(\bar{r}_1, \bar{K})$ which differs from that defined under (3) only by the $\bar{r}_1$ dependence. The scattering dyadic $\bar{\bar{S}}_{22}^3$ represents any scattering target and the scattered field can be considered to have originated from an antenna. As a result, near-field antenna measurement techniques are applicable (reference 2). Denote the coupling-product by

$$D(\bar{K}, \bar{r}_1) = \bar{\bar{I}}_{01}(\bar{K}) \cdot \bar{\bar{I}}_{10}^{\prime}(\bar{r}_1, \bar{K}) \qquad (21)$$

Deconvolution of (21) allows $D(\bar{K}, \bar{r}_1)$ to be expressed as:

$$D(\bar{K}, \bar{r}_1) = \frac{1}{4\pi^2 a_0^1} \int b_0^2 (\bar{r}_1, \bar{r}_2) \; e^{i\bar{K} \cdot \bar{r}_2} \; d\bar{R}_2 \qquad (22)$$

Since for planar near-field antenna measurment techniques b_0^2 represents sampled data in the transverse $\bar{R}_2$ plane, $D(\bar{K}, \bar{r}_1)$ can

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be empirically determined, and $\bar{I}'_{10}(\bar{K},\bar{r}_1)$ can be computed and receive probe corrected via (21). A far-electric field can then be calculated and an RCS evaluated although it will generally be transmit probe dependent. If a plane-wave is incident upon the target then $\bar{I}'_{10}(\bar{K},\bar{r}_1)$ will not be transmit probe dependent, and the far-electric field can be calculated and an RCS determined. However, even if $\bar{I}'_{10}(\bar{K},\bar{r}_1)$ is transmit probe dependent a second deconvolution can be performed upon (19) and $\bar{\bar{S}}^3_{22}(\bar{K},\bar{L})$ evaluated. $\bar{\bar{S}}^3_{22}(\bar{K},\bar{L})$ is the most essential parameter in the PWSM formulation of the RCS problem since it describes the scattering properties of the target.

Defining

$$\bar{Q} = (\bar{K},\bar{L}) = \bar{\bar{S}}_{22}(\bar{K},\bar{L}) \cdot \bar{I}_{10}(\bar{L}) \tag{23}$$

as a scattering product, a second deconvolution can be written as

$$\bar{Q}(\bar{K},\bar{L}) = \frac{1}{4\pi^2} \int \bar{I}'_{10}(\bar{K},\bar{r}_1) \, e^{-i\bar{\ell}\cdot\bar{r}_1} \, d\bar{R}_1 \tag{24}$$

An explicit set of equations, assuming TM(x) and TE(y) polarization, can be written as:

$$Q_x(\bar{K},\bar{L}) = I_{10x}(\bar{L}) \, S^3_{22}(x,\bar{K};x,\bar{L}) + I_{10y}(\bar{L}) \, S^3_{22}(x,\bar{K};y,\bar{L})$$

$$\tag{25}$$

$$Q_y(\bar{K},\bar{L}) = I_{10x}(\bar{L}) \, S^3_{22}(y,\bar{K};x,\bar{L}) + I_{10y}(\bar{L}) \, S^3_{22}(y,\bar{K};y,\bar{L})$$

If the transmitter is rotated and (20) through (25) resolved, $\bar{\bar{S}}_{22}^{3}$ can be determined. Once known, $\bar{\bar{S}}_{22}^{3}$ can be used in the reflection integral (18) allowing b_0^2 to be calculated for any incident field, and an RCS can be determined in the near- or far-field.

6. Discussion

The TSR interaction is compactly expressed in the reduced-reflection integral (18). This integral describes a transmitted radiation pattern which is propagated to a target and scattered. Emerging from this interaction is another, re-transmitted radiation pattern. This target radiation pattern is determined by evaluating and summing the scattering-product (23) for all incident directions. The target radiation pattern is then propagated to and received by a probe antenna which also has a particular pattern (or angular spectrum). The first deconvolution (22) allows the coupling-product (21) to be evaluated, which in turn enables the transmit-target pattern to be determined and receive probe corrected. A second deconvolution (24) allows the scattering-product to be evaluated which in turn enables the dyadic scattering tensor element to be determined and transmit probe corrected.

The scattering tensor element S_{22}^{3} $(m,\bar{K};n,\bar{L})$ is a quantity which enforces the electromagnetic boundary conditions to be satisfied for any of the incident field directions $\bar{L}$ and polarizations n. This enforcement of the boundary conditions is dependent upon the target geometry and electromagnetic

constitutive parameters σ, μ and ε. As such, the dyadic scat-
tering tensor element is a significant quantity for the radar
problem since it inherently contains the target geometry and
electromagnetic constitutive parameters. Since the scattering
tensor is independent of transmit and receive probes it can be
used as a classification parameter for various targets and may
also be used for calculating a measured RCS using any set of
probes, in the near- or far-field.

A final note to be mentioned is that if the first decon-
volution required an $N \times N$ array of data, then the second decon-
volution would require an $(N \times N)^2$ array for determining S_{22}^3
$(m,\bar{K};n,\bar{L})$. Efficient data acquisition and processing schemes are
needed to minimize computation memory and time requirements.

7. Conclusion

A three-antenna PWSM formulation has been presented and a
solution formally obtained. The radar problem or TSR interaction
is a special case of the three-antenna problem. A transmission
integral was obtained (20) which is similar to the one obtained
in reference 1. This substantiates using near-field antenna
measurement techniques for measuring the near-scattered field of
a target. The scattered field transmission pattern can be deter-
mined and probe corrected as in the near-field antenna measure-
ments. A near- or far-field electric field and corresponding RCS
can then be calculated. This RCS will in general be probe-
dependent. However, applying the second deconvolution (24)

allows the scattering-product to be evaluated and the target scattering parameters determined and transmit probe corrected. Using an arbitrary incident field, a near- or far-field electric field and the corresponding RCS can be calculated.

Future efforts include the following topics:

1. Analytically calculating the dyadic scattering tensor elements from known scattered field solutions.

2. Performing a similar three-antenna analysis with one of the antennas located in a plane mutually orthogonal to the other two.

3. Determining how to simulate a "near-field" plane-wave and thus avoid a full second deconvolution.

4. Fully utilizing a given near-field measurement data-set, including simulating with software other incident field directions.

References

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